ERRORS IN STEREO DUE TO QUANTIZATION

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MS-CIS-85-34
GRASP LAB 47

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September 1985

Acknowledgements: This research was supported in part by DARPA grants NOOO14-85-K-0018 and NOOO14-85-K-0807, NSF grants DCR-86-07156, DCR8501482, MCS8219196-CER, MCS-82-07294, 1 RO1-HL-29985-01, U.S. Army grants DAA6-29-84-K-0061, DAAB07-84-K-F077, U.S. Air Force grant 82-NM-299, AI Center grants NSF-MCS-83-05221, U.S. Army Research office grant ARO-DAA29-84-9-0027, Lord Corporation, RCA and Digital Equipment Corporation.

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ABSTRACT

Quantization errors in the stereoptic method due to discrete photoelements in cameras are analysed and their relations to the distance between the object and the cameras (range), distance among the cameras (baseline) and the focal length of the camera lenses are given.

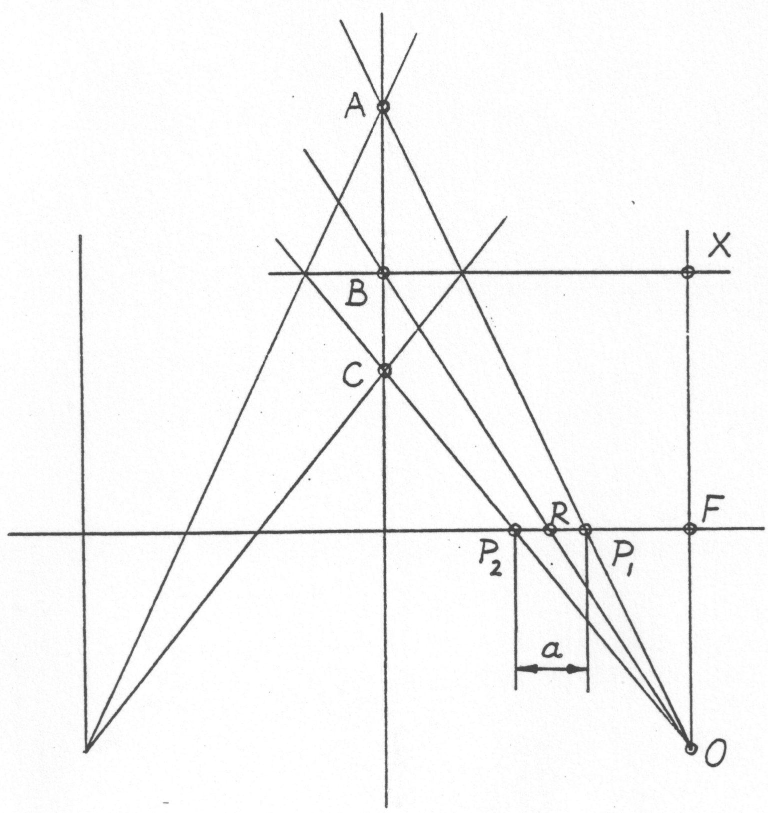
Introduction

Stereoscopy is a method using two cameras from two positions for computing depth information. There has been a considerable effort to use it in computer vision (Ballard and Brown 1982). Stereoscopic calculation involves two separate steps. First the image point in each pictures corresponding to the object point must be found. This correspondence problem is the hardest part of the stereoscopic method and although many solutions have been proposed there is still no general method which will perform well in all circumstances. The rest is relatively easy. A simple trigonometric calculation must be performed to determine the intersection of the two projecting rays to get the position of the world point.

Since digital cameras are used in most of the stereoscopic experiments in computer vision, the influence of the quantization should be considered because the size of picture elements (pixels) is not negligable (Duda and Hart 1973). The extent of errors due to pixel size, focal length, base line and most of all the distance from the cameras will be investigated.

Quantization errors

Let us imagine an experiment to investigate the effect of picture quantization on the stereo triangulation accuracy. Two pictures of a single object point are taken with two different fixed cameras. The cameras are mounted on a baseline with parallel optical axes (Fig. 1). Let each of the pictures be quantized, so that the true image points are replaced by the center points of the grid cells in which they fall (points P in Fig. 1). The object point computed from the quantized image points will in general be different from the original object point. In the plane defined by the optical axes of both cameras the rays projecting from the focal points of the cameras through the centers of photo elements form diamond shaped regions (Fig. 1).



BX = baseline/2
OX = range
f = OF = focal length of lenses
a = P1P2 = horizontal center to center distance between photo elements

Fig. 1: Points that lie between A and B are projected into pixel P1 in the quantized image, while points between B and C project to P2.

The diamond shaped regions grow with the distance from the cameras as can be seen in Fig. 2. All points in a particular region correspond to a single combination of two pixels in the images. Hence when reconstructing the world points, the location of a world point can be determined only up to the location of the corresponding diamond shaped region.

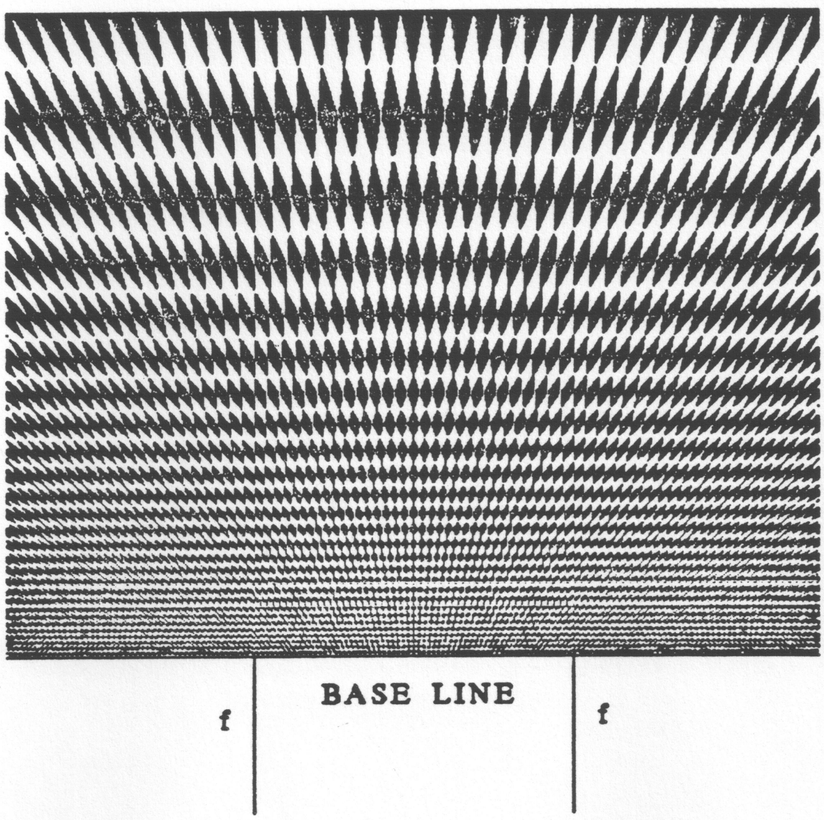


Fig 2: The plane in front of a pair of stereo cameras is subdivided into diamond shaped regions. (For explanation see text.) In this simulation the focal length of the cameras was 33 pixels and of the base line 66 pixels.

One of the measures of the difference between the computed distance and the real distance is the percentage range error, where range is defined as the distance from the object point to the midpoint of the baseline connecting the focal points of both lenses. It is intuitively clear that the percentage range error will typically increase as the true range increases, for as the true range increases the projecting rays become more nearly parallel and small errors are likely to have severe consequences. For simplicity assume that the image point lies on the line perpendicular to the midpoint of the baseline. Imagine now that the object point is moved further and further away from the cameras. As the point recedes, its image in the quantized picture moves from pixel to pixel, being first to the left and then to the right of the true image. This makes the projecting rays first too nearly parallel and then too convergent (Fig. 1). Consequently, the computed location of the object point is first too far away and then too near.

When the object point recedes further away the same process is repeated for the next quantization cell. As a result we get a serrated error curve as shown in Fig. 3 which is different for each focal length of the lenses. The error can be computed easily due to the similar triangles seen in Fig. 1:

$$RF = \frac{BX \; . \, f}{XO}$$

RF is the distance from the true image point to the optical axis of the left camera. Since the picture is quantized the true image point is substituted by the nearest grid cell which is P1 or P2 in Figure 1. The error measured on the line perpendicular to the baseline is equal to lenght BA if R is nearer to photoelement P1 and to BC if R is nearer to P2. The percentage range error is hence given by:

$$\text{if } (RF - P_1F) > \frac{a}{2} \text{ then } error = \frac{OF \; . \, BX}{FP_2} - 1$$

$$\text{if } (RF - P_1F) < \frac{a}{2} \text{ then } error = \frac{OF \; . \, BX}{FP_1} - 1$$

Note also that the range error is the same for points off the central axis as can be seen in Fig. 2. Errors in the plane perpendicular to the optical axis, however, grow when the object point moves away from the central axis and become larger than the range error when the ray from the camera to the object point encloses less than a 45 degree angle with the base line.

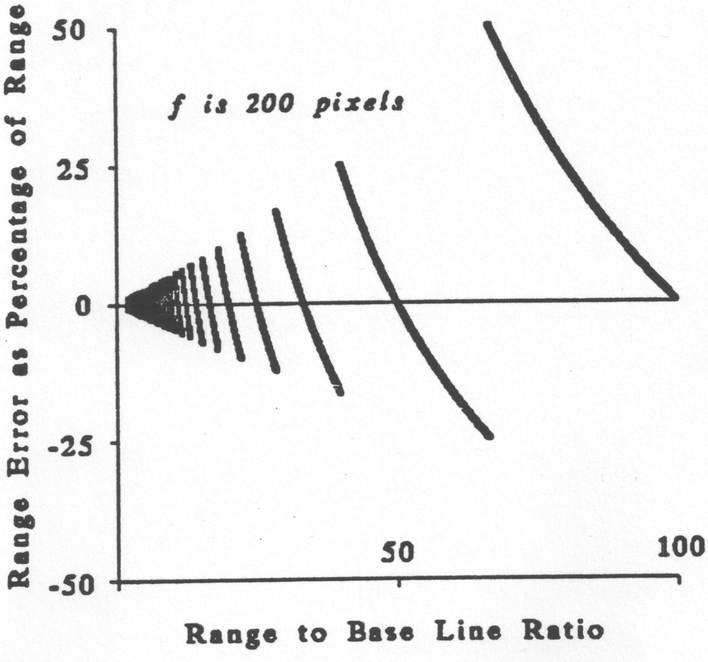


Fig. 3: Result of the stereopsis experiment. Range errors grow with range to base line ratio. The focal lenght of the lenses is given in pixels because the error depends only on the ratio of the focal lenght to the pixel width.

The cameras used in the GRASP Laboratory have the following characteristics:

Type of cameras: Fairchild, model PS-3000

Type of lenses: Fujinon TV zoom lens, model C6 x 17.5B

Area image sensor: CCD 222
with 488 X 380 elements,
dimensions of the photoelements are 12um horizontal by 18um vertical,
photoelements are positioned on 30um horizontal centers
and 18um vertical centers;

Focal length of lenses: 17.5 - 105 mm

Minimum object distance: 1.3 m

Aperture ratio: 1 : 1.8

The cameras are mounted on a platform with a baseline of 12.7 cm (5 inches).

By substituting the appropriate values for the two cameras in the GRASP Laboratory into the above equations we get the following figure:

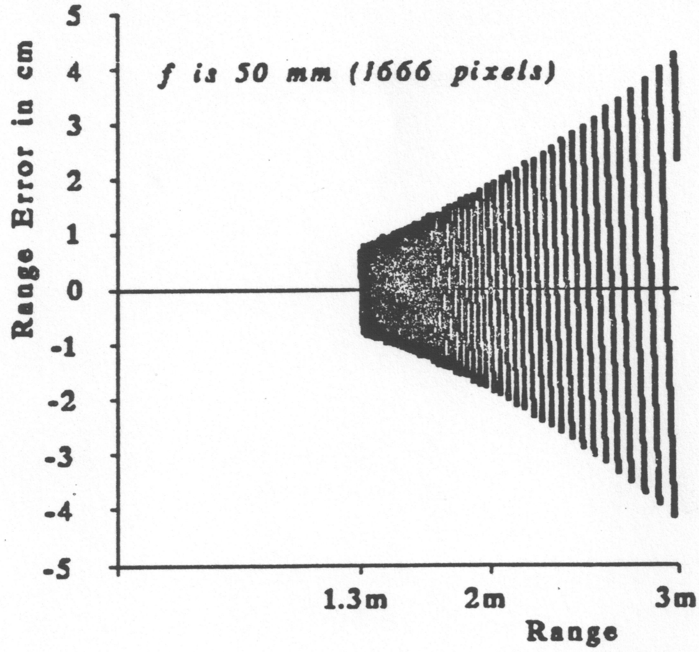


Fig. 4: Range errors as a function of range for the cameras in the GRASP Laboratory.

In the previous case we assumed a constant focal length for the camera lenses. How does a changing focal length influence the range error if the distance to baseline ratio remains constant? Using the same equations derived from Figure 1 and considering only the worst case error for each focal length (that amounts to depicting the envelope of the serrated function) we get the following equation for the relative range error:

$$error = \frac{1}{1 - \dfrac{range \cdot \frac{a}{2}}{f \cdot \frac{baseline}{2}}} - 1$$

Range errors hence get smaller with longer focal lengths as can be seen in Fig. 5, where the focal range corresponds to the GRASP cameras.

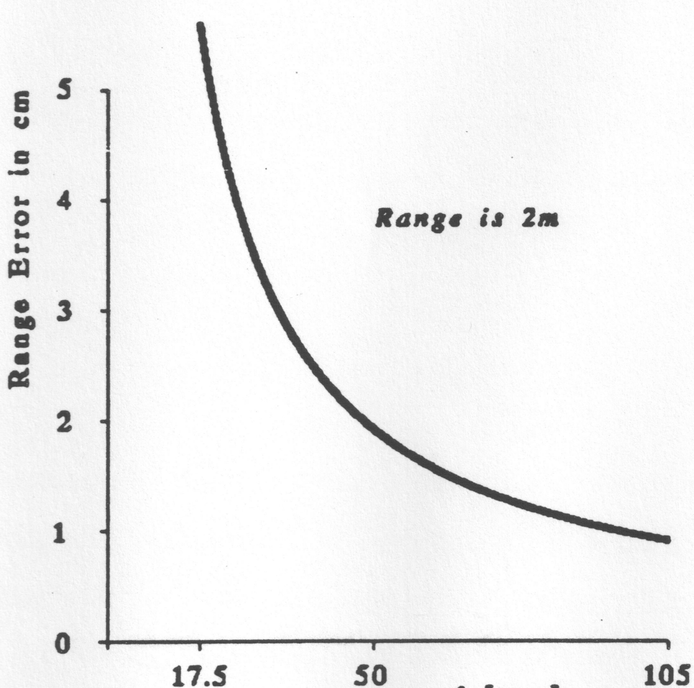


Fig. 5: Worst case range error as a function of focal length. The range is a constant.

To reduce the range error, the use of longer focal lengths is sensible only up to a limit. This is because longer focal lengths depict smaller parts of the scene and hence the overlapping viewfield seen from both cameras lies further away (assuming the optical axes of the cameras stay parallel and the length of the base line stays the same). Other problems with long focal lengths are that the depth of the scene in focus sharply decreases and mechanical vibrations become a problem.

Errors due to movement of the camera platform

So far we have assumed that the cameras remain fixed during our experiment. The cameras in the GRASP Laboratory are mounted on a specially built platform (Summers 1984) that gives them four degrees of freedom: up and down, side to side, pan motion and tilt motion. The platform position and the update of the platform position is controled with a 8085 based microprocessor which is interfaced to a VAX 11/750 so that programs running on the VAX can control the movement of the platform. Due mostly to the mechanical construction, the cameras do not always stop in the desired position. The platform was thoroughly tested for positional accuracy (Summers 1984) and the following standard deviations were recorded:

horizontal axis	:	1.13 units	=	0.20 cm
vertical axis	:	2.49 units	=	0.33 cm
pan axis	:	2.27 units	=	0.77 degrees
tilt axis	:	3.32 units	=	0.93 degrees

When the reconstructed world points from different positions are to be combined - for example when building a complete 3-D representation of an object - the positional errors should be considered. While the errors in horizontal and vertical axis must be accounted for, they do not get amplified with the distance from the cameras to the observed object as do the pan and tilt errors. When the platform is moved through several positions starting from a reference position the errors propagate. A rough estimate of the worst case error in each consecutive position is the sum of all previous errors.

Assume that the platform is in a reference position. After a move to a new position the cameras will be misplaced in the X-Y plane up to:

$$e = \sqrt{e_{horizontal}^{2} + e_{vertical}^{2}} = 0.59\ cm$$

and pointing away from the desired direction up to:

$$angle = arctg \sqrt{(tg\ pan)^2 + (tg\ tilt)^2} = 1.2\ degrees$$

As the angle is small, the range error (error in Z direction) is negligable, while the worst case error in the plane perpendicular to the optical axis (X-Y plane) is the sum of error *e* from the above equation and the tilt and pan error:

$$error = e + range\ .\ tg(angle)$$

This error depends on the distance of the reconstructed point from the cameras (Fig. 6).

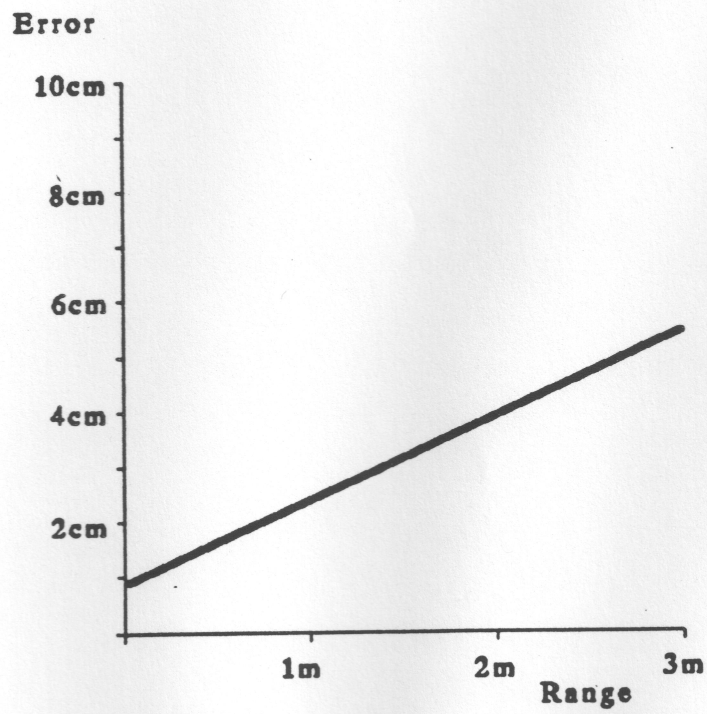


Fig. 6: Errors in position in the plane perpendicular to the optical axis due to the positional inaccuracies.

Conclusions

The findings about quantization errors in the stereoptic method and their relation to the distance between the object and the cameras, to the distance among the cameras (baseline) and to the focal length of the camera lenses are given. This findings should help to establish a more controlled environment for experiments with stereo cameras.

It is important to notice that there exist inherent limits as to how accurate the stereoptic method can be. Quantization errors are always present and even the use of better equipement and finer grids of photo elements can not eliminate them entirely.

Some simple strategies for working with the stereo method are to put the investigated objects closer to the cameras and to use longer focal lengths if possible. The right way to deal with the errors is to use better software methods. In some cases, when the world objects have straight lines, interpolation can be used. Better accuracy can also be achieved by taking multiple stereo pairs of pictures. Due to the positional errors, however, one can not just paste together the reconstructed 3-D points from each stereo pair of pictures. One has to solve the correspondance problem among these sets of 3-D points to get a unique 3-D representation of the scene.

For a stereo measurement synthesis procedure to meet given specifications see McVey and Lee.

REFERENCES

Ballard D.H., Brown C.M.: *Computer Vision*. Englewood Cliffs, N.J., Prentice-Hall, 1982.

Duda O.R., Hart P.E.: *Pattern Classification and Scene Analysis*. New York, Wiley, 1973.

McVey E.S., Lee J.W.: *Some Accuracy and Resolution Aspects of Computer Vision Distance Measurements*, IEEE Trans on PAMI-4, NO. 6, pp. 646-649, 1982.

Summers J.F.: *Final Report: The Design and Construction of a Four Degree of Freedom Camera Controller*, MS-CIS-84-52, GRASP LAB 24, University of Pennsylvania, Philadelphia, 1984.